

Skeptical review: The Turbulent Architecture and Convective Drivers of Mass Transfer in a Red Supergiant Binary

Summary

The manuscript presents a detailed, multi-diagnostic analysis of a single 3D radiation-hydrodynamics Athena++ snapshot of a red-supergiant donor undergoing Roche-lobe overflow. The authors (i) reconstruct an effective Roche potential and mask material outside the L1 equipotential (Sec. 2.1–2.2), (ii) quantify the morphology and intermittency of the mass-flux field and show a strongly filamentary, bursty outflow rather than a smooth stream (Sec. 3.1–3.2, Table 2, Fig. 2), (iii) apply the Q-criterion to assess flow topology and argue the L1 stream is shear-dominated without large coherent vortices (Sec. 2.3, Sec. 3.3, Fig. 4), and (iv) introduce an upstream “source fingerprinting” procedure tracing stream cells back to a photospheric surface and comparing the resulting footprint’s properties to the global surface (Sec. 2.5, Sec. 3.4, Table 3, Fig. 5). The headline interpretation is that localized convective upwellings (lower density/pressure, higher outward radiation flux, more positive radial velocity) preferentially supply material to the L1 stream. The overall narrative is compelling and the analysis toolkit is potentially valuable, but several conclusions are currently overextended given the reliance on a single snapshot and the under-specification/validation of key threshold-based steps (Roche masking, Q-structure identification, and source tracing). The paper would be substantially strengthened by (a) clearer documentation of the underlying simulation and gravity/potential consistency, (b) robustness checks across nearby snapshots and parameter/threshold choices, and (c) quantitative presentation of the force/work decomposition that underpins the “driving engine” claims (Sec. 2.4, Sec. 3.4, Sec. 4).

Strengths

- Targets an important and difficult regime—RLOF from convective red supergiant donors—with clear motivation and potentially high relevance to binary-evolution prescriptions (Sec. 1, Sec. 4).
- Presents a coherent, layered analysis pipeline (Roche-geometry masking, mass-flux statistics, Q-criterion topology, force/work decomposition, and upstream source tracing) applied consistently to the same dataset (Sec. 2).
- Provides strong evidence (within this snapshot) that the outflow is highly filamentary and intermittent, supported by spatial maps and distributional statistics (Sec. 3.1–3.2; Fig. 2; Table 2).
- The “source fingerprinting” concept is intuitive and, if robust, could become a broadly useful diagnostic for linking mass-loss/mass-transfer streams to localized surface conditions (Sec. 2.5; Sec. 3.4; Table 3; Fig. 5).
- Generally clear organization and readable presentation; figures (notably Figs. 2, 4, 5) aim to connect morphology, statistics, and physical interpretation in an accessible way.
- Core definitions for mass flux, velocity-gradient decomposition, and Q-criterion are standard and internally consistent (Sec. 2.3).
- Force-density and power-density definitions are dimensionally consistent as written and are a good framework for a localized energy-injection/acceleration analysis (Sec. 2.4).

Major issues

1. **Overgeneralization from a single snapshot: the manuscript frequently uses definitive/causal language (e.g., “definitive,” “primary engine,” “direct causal link,” “dictates the entire filamentary architecture”) while the analysis is based on one instantaneous snapshot of an intrinsically intermittent convective/turbulent system (Abstract; Sec. 1; Sec. 3; Sec. 4).** Without temporal context, it is unclear how representative the reported filamentarity, extreme intermittency (very large kurtosis), Q-structure conclusions, and footprint contrasts are.

Recommendation: Recalibrate claims throughout (Abstract; Sec. 1; Sec. 3; Sec. 4) to explicitly frame the work as a detailed case study of one snapshot (“in this model snapshot...”, “suggests...”, “is consistent with...”). If feasible, add a minimal multi-snapshot robustness check (even 3–5 snapshots bracketing the analyzed time): show time variability of (i) a mass-transfer proxy (e.g., flux through an L1 control surface or across $\Phi = \Phi_{L1}$), (ii) key PDF moments (mean/variance/kurtosis of $|j|$) in global and L1 regions (Sec. 3.2), and (iii) footprint area/fraction and Table 3 mean ratios (Sec. 3.4). At minimum, report where the snapshot sits in the simulation timeline (time since RLOF onset; fractions of orbital/convective turnover times) and discuss representativeness explicitly in Sec. 4.

2. **Insufficient documentation of the simulation and gravity/potential consistency limits reproducibility and interpretation.** Sec. 2.1 mentions Athena++ but omits key physical/numerical parameters (M1, M2, q, separation/period, rotation state, donor radius/luminosity/ T_{eff}), EOS, opacities, radiation closure/transport, resolution, domain extents, boundary conditions, companion treatment/sink, runtime).

Moreover, the analysis builds an effective Roche potential (Sec. 2.1–2.2), but it is unclear whether this potential matches the gravity actually used in the simulation (e.g., point-mass vs self-gravity/monopole/multipole, indirect terms, softening). In an extended, non-spherical donor, “outside Roche lobe” can be sensitive to these choices.

Recommendation: Expand Sec. 2.1 with a concise but complete simulation summary: system parameters (M_1 , M_2 , q , a , P , Ω), donor properties, rotation/corotation assumptions, grid (n_r , n_θ , n_ϕ), extents, boundary conditions, EOS, radiation method (e.g., FLD/M1; comoving vs lab-frame moments), opacity treatment, gravitational softening/sink/accretion prescription, and evolution time to the analyzed snapshot. Clarify explicitly what gravitational potential the simulation evolves under, and use that same potential field in the Roche-mask analysis if available. State how Φ_{L1} and the L1 location are found numerically (e.g., 1D search along the line of centers vs saddle search), and report x_{L1} and Φ_{L1} (Sec. 2.1–2.2). Add a short sensitivity test showing how the “outside Roche lobe” mask fraction changes under modest perturbations of Φ_{L1} (or equivalent threshold).

3. **The “source fingerprinting” method (Sec. 2.5) is central to the paper’s main claim (convective upwellings preferentially supply the stream), but the algorithmic details and robustness are not sufficient to support causal/exclusive phrasing.** It is unclear whether trajectories are true streamline integrations through the 3D field or a single back-projection; the seeding layer “just outside the Roche lobe” is not precisely defined; and failure cases (non-intersection with the surface, leaving domain) and interpolation/step-size errors are not discussed. In a strongly accelerating, curved, compressible flow near L1, a simplistic back-projection can misidentify origins.

Recommendation: Substantially expand Sec. 2.5 with operational details: (i) define seeding cells (exact radial range/thickness relative to $\Phi = \Phi_{L1}$ and any $v_r > 0$ or $|j|$ thresholds), (ii) specify whether you integrate $dx/ds = -v(x)$ (and which integrator, step size/adaptive control, interpolation scheme), or do a one-step projection; (iii) list termination criteria and how you treat paths that do not hit the photospheric surface or exit the domain; (iv) report the traced mass fraction (or number of stream cells) that successfully maps to the surface. Add robustness tests: vary seeding-layer thickness and photospheric radius (± 5 – 10%) and show Table 3 ratios/footprint morphology are stable (Sec. 3.4). Replace “direct causal link”/“exclusive” with “strong association consistent with...” unless additional validation is provided (e.g., forward–backward consistency checks, or time-integrated tracer particles/passive scalars if available). Quantify overlap with objectively defined upwellings (e.g., v_r percentile threshold) via contingency tables/odds ratios or KS tests on v_r and $F_{r,rad}$ distributions (Sec. 3.4).

4. **Force/work decomposition is described (Sec. 2.4) and invoked to argue that convection plus gas/radiation pressure lift material over the barrier (Sec. 3.4; Sec. 4), but the manuscript does not present the quantitative outcomes needed to support relative-importance claims (e.g., gas vs radiation vs gravity; where positive work is done).** As written, the central “driving engine” conclusion is not adequately substantiated.

Recommendation: In Sec. 3.4, add quantitative conditional statistics for forces and power densities: PDFs and/or mean/median ratios of $\frac{F_{gas}}{F}$, $\frac{F_{rad}}{F}$, and W_{gas} , W_{rad} , W_{grav} in (a) fingerprintsur facecells, (b) non – fingerprintsur facecellsonthefacinghemisphere, and (c) theL1streammask. Consideralsoreportingsignedwork(howoften $W_{rad} > 0$, $W_{gas} > 0$)todemonstratesystematicaccelerationvsrandomfluctuations. Clarifythat $\frac{F_{rad}}{F} = -\nabla \cdot P$ is computed consistently with the simulation’s radiation closure/frame (Sec. 2.4). Use these results to refine (and if necessary soften) claims in Sec. 4 about which force actually dominates the launching/acceleration in this model.

5. **Roche potential equation and reference-frame specification are currently ambiguous/problematic, undermining the Roche-mask and Φ_{L1} definitions (Sec. 2.1–2.2).** The Roche-potential expression appears dimensionally inconsistent as written (companion term formatting) and the centrifugal term is not explicitly defined with respect to origin/axis; if donor-centered coordinates are used, indirect terms/offsets must be handled consistently.

Recommendation: Rewrite the Roche potential in a fully parenthesized, standard form with explicit coordinate dependence and clearly defined origin and rotation axis (barycentric vs donor-centered rotating frame). State whether indirect terms are included and ensure consistency with how Φ_{L1} is computed and how the simulation’s rotating frame is defined (Sec. 2.1). After rewriting, verify that the “outside Roche lobe” criterion ($\Phi_R > \Phi_{L1}$) matches the stated sign convention and yields the intended geometry (Sec. 2.2).

6. **Q-criterion-based claims (“no stable, long-lived vortices,” “developed turbulence,” “shear-dominated”) are based on a single snapshot and a largely threshold-based, qualitative connected-component analysis (Sec. 2.3; Sec. 3.3; Fig. 4).** For compressible, stratified, shearing flows, Q alone can be subtle to interpret; results may depend strongly on Q_{thresh} , resolution, derivative noise, and whether structures are identified in true 3D connectivity or inferred from slices.

Recommendation: In Sec. 2.3 and Sec. 3.3, (i) report Q distribution statistics (percentiles or mean/ σ) and reconcile any inconsistent descriptions of thresholding; (ii) perform a threshold-sensitivity scan (e.g., vary Q_{thresh} over 1–2 dex or in units of σ_Q) and show how the number/volume of connected positive- Q structures changes; (iii) state clearly the 3D connectivity definition (e.g., 6/18/26-neighbor equivalent), minimum voxel count, any smoothing/filtering, and masking (Sec. 3.3). If feasible, add one complementary diagnostic (e.g., $|\omega|$, enstrophy, or λ_2) to support the “shear-dominated” interpretation. Temper conclusions to “no large coherent vortices above resolution/threshold limits in this snapshot” unless temporal persistence is demonstrated across multiple outputs.

7. **Mass-flux intermittency statistics may be distorted by sampling choices and numerical floors.** The extremely large kurtosis (Sec. 3.2) can reflect genuine intermittency, but can also be amplified by inclusion of near-vacuum/background regions, mixing inflow/outflow, cell-volume weighting choices, and pile-ups at density/pressure floors (Table 1). The “global vs L1 region” comparison is hard to interpret because the L1 region geometry is not precisely defined (Sec. 2.2; Sec. 3.2; Table 2).

Recommendation: In Sec. 2.2 and Fig. 2 caption, explicitly state weighting (cell-count vs volume vs mass), binning, whether ghost zones are excluded, and how floors are treated (masking/marking; report fraction of cells at floors). Provide restricted PDFs (or at least key moments) for physically relevant subsets: $v_r > 0$ outflow, outside-Roche mask, and/or within a donor-neighborhood radius, to show intermittency is tied to launching/stream rather than ambient low-density volume. Precisely define the L1 “localized spherical region” (center coordinates and radius) used in Table 2 and demonstrate modest robustness to that radius choice.

Minor issues

1. Figures and captions (especially Figs. 2, 4, 5) lack key “reader contract” details: PDF normalization and weighting, sample size/bins, units or explicit ‘code units’, and clear marking of L1/Roche geometry and viewing orientation (Sec. 3.1–3.4). Fig. 4’s histogram panel is described as incomplete/unclear (axes/units/binning).

Recommendation: Revise captions to specify normalization (PDF vs counts), weighting, binning, sample size, and whether results are single-snapshot or time-averaged. Add explicit units or state “code units” consistently and provide key conversions/scales where helpful. Overlay/annotate Roche lobe and L1 location on relevant panels and clarify viewpoint/orientation. Ensure Fig. 4 histogram has labeled axes/units, bin edges, and region selection (global vs L1) clearly stated.

2. Definition/notation gaps in radiation quantities: Table 3 and discussion use “radial radiation flux ($F_{r,\text{rad}}$)”, but Methods focus on radiation pressure tensor $\mathbf{P}(r)$ and its divergence (Sec. 2.4; Sec. 3.4). Also, the physical interpretation of “low radiation energy E ” in the footprint is not discussed, but higher outward $F_{r,\text{rad}}$

Recommendation: Define $F_{r,\text{rad}}$ explicitly in Sec. 2.4 or Sec. 2.5 (source field, sign convention, frame) and ensure consistent notation for $\mathbf{P}(r)$ (tensor notation and index convention). Add 2–3 sentences in Sec. 3.4 interpreting the E contrast (e.g., anisotropy/streaming, opacity or gradient effects) or note it as an open diagnostic outcome, vs $F_{r,\text{rad}}$

3. Finite-difference/derivative methodology is under-specified for quantities sensitive to numerical noise (velocity gradients for Q ; $\nabla \cdot \mathbf{P}_r$ for radiation force; pressure gradients) in spherical coordinates (Sec. 2.3–2.4).

Recommendation: Add a brief methodological paragraph (or appendix) specifying stencil order, handling of spherical metric terms, polar axis treatment, boundary derivatives, and any smoothing/filtering. If possible, add a small internal check (e.g., repeating Q statistics with a slightly different stencil or mild smoothing) to demonstrate qualitative robustness.

4. The photospheric/surface definition used for source fingerprinting ($r \approx 597R_\odot$) is introduced late (Sec. 3.4) without prior definition/justification (Sec. 2.5). It is unclear whether this corresponds to an optical-depth surface or a fixed geometric shell, and what the shell thickness is in physical units.

Recommendation: Move the surface definition to Sec. 2.5: how $r = R_{\text{RSG}}$ is chosen ($\tau \approx 1$, density/pressure contour, or fixed radius), shell thickness (one cell? a band?), and why it is appropriate for “photospheric origin.” In Sec. 3.4, report sensitivity of Table 3 contrasts to modest changes in the adopted radius (or cite the robustness test requested under Major Issue #3).

5. Related-work and novelty framing could be sharper. Sec. 1 motivates the topic well but gives limited quantitative/citation context for prior 3D RLOF/giant convection studies and for existing “source tracing” approaches (streamlines, passive scalars, tracer particles).

Recommendation: Expand Sec. 1 with a concise paragraph situating this work relative to prior multi-D RLOF and convective-envelope simulations, and clarify what is new here (the combination of diagnostics; instantaneous filament/intermittency quantification; surface-to-stream statistical linkage). In Sec. 2.5/Sec. 4, briefly compare “source fingerprinting” to established tracer/streamline methods and describe it as an adaptation/extension unless clear novelty is demonstrated.

6. Reproducibility and data/code availability are only partially addressed (Sec. 2). Several analysis steps (connected-component labeling; tracing; masking) are nontrivial to reimplement from the current description.

Recommendation: Add a short reproducibility statement (end of Sec. 2 or in Sec. 4): whether analysis scripts and key derived products (masks, structure catalogs, tables) will be made public, with a repository link if available. Even if code cannot be released, summarize key algorithmic parameters (connectivity, thresholds, filters) in a table for reimplementa-tion.

7. Interpretation tends to attribute the stream architecture almost entirely to convection and pressure forces, with limited explicit discussion of tidal/rotational funneling and large-scale shear that are intrinsic to RLOF geometry (Sec. 3.4; Sec. 4).

Recommendation: Add a short balancing paragraph in Sec. 3.4 or Sec. 4 acknowledging how tides/rotation set the large-scale L1 nozzle and shear, while convection/pressure fluctuations modulate which parcels reach/overcome the effective potential barrier in this model. Tie this explicitly to the quantitative force/work results once added.

Very minor issues

1. Numeric/text consistency: the statement that the L1-region mean flux is “almost 20 times higher” than the global mean conflicts with Table 2 values ($0.501464/0.028256 \approx 17.747$), which is outside a 10% tolerance.

Recommendation: Revise to “ $\approx 17.7\times$ ” / “ $\approx 18\times$ ” or adjust the comparison to match the computed ratio.

2. Kurtosis convention is not stated (Pearson vs excess), yet the text references “Gaussian kurtosis = 3” (Sec. 3.2).

Recommendation: State explicitly which definition is used and ensure comparisons in the text match that convention.

3. Minor typographical/formatting issues reduce polish: stray line breaks (e.g., in Sec. 1), inconsistent capitalization (“Roche Lobe Overflow” vs “Roche lobe overflow”), inconsistent quotation/term styling (“source fingerprint”), and HTML escape sequences (e.g., “>”) in math (Sec. 1; Sec. 3.3; Sec. 4).

Recommendation: Proofread for line breaks, standardize capitalization/terminology, ensure LaTeX renders inequalities properly, and split a few overly long sentences in the Abstract and Sec. 4 for readability.

4. Minor notation clarity: tensor notation for radiation pressure P_r is sometimes not distinguished typographically, and the divergence index convention is not stated (Sec. 2.4).

Recommendation: Use consistent tensor notation (e.g., \mathbf{P}) and optionally add the component form $(\nabla_{\mathbf{a}} \cdot \mathbf{P})_i = \partial_j P_{ij}$ in the chosen coordinates.

Mathematical consistency audit

This section audits **symbolic/analytic** mathematical consistency (algebra, derivations, dimensional/unit checks, definition consistency).

Maths relevance: light

The paper contains a small set of central analytic definitions (coordinate transform, Roche potential, mass flux, Q-criterion, force densities, and power density) used to construct masks/diagnostics on simulation data. There are few step-by-step derivations; most mathematics is definitional and relies on correct frame/notation choices.

Checked items

1. ✓ **Spherical-to-Cartesian transform** (Eqs. (1)–(3), Sec. 2.1, p.2)
 - **Claim:** Defines $X = r \sin \theta \cos \phi$, $Y = r \sin \theta \sin \phi$, $Z = r \cos \theta$ for mapping the spherical grid to Cartesian coordinates.
 - **Checks:** algebra, notation consistency
 - **Verdict:** PASS; confidence: high; impact: minor
 - **Assumptions/inputs:** θ is the polar (colatitude) angle and ϕ is the azimuthal angle in the simulation’s spherical grid.
 - **Notes:** Standard coordinate relations; no internal conflicts elsewhere in the paper.
2. ✗ **Roche potential: companion gravitational term appears dimensionally wrong** (Unnumbered equation for Φ_R in Sec. 2.1, p.2)
 - **Claim:** Defines the effective Roche potential as the sum of two point-mass gravitational potentials and a centrifugal potential term.
 - **Checks:** dimensional/units consistency, notation/parentheses consistency
 - **Verdict:** FAIL; confidence: medium; impact: critical

- **Assumptions/inputs:** Φ_R is intended to have units of specific potential energy (e.g., cm^2s^{-2} in cgs)., Distances are measured from the donor-centered Cartesian coordinates defined by Eqs. (1)–(3).
 - **Notes:** As written in the provided PDF text, the companion term appears as “ $-\sqrt{GM_2}/((X-D)^2 + Y^2 + Z^2)$ ” (or equivalently with the square-root only over GM_2 rather than over the distance). This is not dimensionally consistent with a gravitational potential term, which should scale like $-GM_2/\sqrt{(X-D)^2 + Y^2 + Z^2}$. The missing/incorrect square-root placement (or missing parentheses) makes Φ_R ambiguous/incorrect in the document as provided.
3. **△ Roche potential: frame/origin consistency of centrifugal term** (Same Roche potential definition, Sec. 2.1, p.2)
- **Claim:** Uses $-(1/2)\Omega^2(X^2 + Y^2)$ as the centrifugal potential while stating the domain is centered on the donor.
 - **Checks:** definition consistency, missing assumptions check
 - **Verdict:** UNCERTAIN; confidence: low; impact: critical
 - **Assumptions/inputs:** The stated Cartesian coordinates (X, Y, Z) are centered on the donor star (Sec. 2.1)., The binary rotates with angular velocity Ω in a co-rotating frame.
 - **Notes:** If (X, Y, Z) are donor-centered, the rotation axis for the centrifugal potential is not necessarily through the donor (unless the donor is at the barycenter). The paper does not specify whether the centrifugal term is computed about the barycenter or about the donor-centered origin, nor does it include any offset/linear inertial terms that may be needed in a non-barycentric rotating frame. Without an explicit frame definition, it is not possible to verify that the Φ_R used to compute Φ_{L1} and the Roche-lobe mask is self-consistent.
4. **✓ Roche-lobe escape criterion** (End of Sec. 2.1, p.2)
- **Claim:** Defines a boolean mask `is_outside_RL` true where $\Phi_R > \Phi_{L1}$ to identify material outside the donor Roche lobe.
 - **Checks:** logical consistency, sign convention sanity check
 - **Verdict:** PASS; confidence: medium; impact: moderate
 - **Assumptions/inputs:** Φ_{L1} is the value of Φ_R at the L1 saddle point., The potential convention is such that ‘higher’ equipotentials correspond to larger Φ_R (less negative).
 - **Notes:** Given the paper’s own convention of negative gravitational terms, selecting $\Phi_R > \Phi_{L1}$ to indicate crossing the L1 equipotential is consistent. This pass is conditional on Φ_R being correctly defined and computed.
5. **✓ Mass flux definition** (Secs. 2.2 and 3.2, pp.3–5)
- **Claim:** Defines mass flux vector $\vec{j} = \rho\vec{v}$ and magnitude $j_{\text{mag}} = |\vec{j}|$ for morphology and PDF statistics.
 - **Checks:** dimensional/units consistency, notation consistency
 - **Verdict:** PASS; confidence: high; impact: minor
 - **Assumptions/inputs:** ρ is mass density; \vec{v} is velocity.
 - **Notes:** Standard definition; no internal contradictions.
6. **✓ Velocity gradient decomposition** (Sec. 2.3, p.3)
- **Claim:** Defines $J = \nabla\vec{v}$, $S = \frac{1}{2}(J + J^T)$, $W = \frac{1}{2}(J - J^T)$.
 - **Checks:** algebra, definition consistency
 - **Verdict:** PASS; confidence: medium; impact: moderate
 - **Assumptions/inputs:** J^T denotes transpose in the chosen basis., $\nabla\vec{v}$ is computed with the appropriate metric terms in spherical coordinates (as stated).
 - **Notes:** Algebraic definitions are correct. The correctness of ‘including all metric terms’ is implementation-dependent and not verifiable from the paper, but the symbolic decomposition is consistent.
7. **✓ Q-criterion formula** (Secs. 2.3 and 3.3, pp.3–6)
- **Claim:** Defines $Q = \frac{1}{2}(\|W\|_F^2 - \|S\|_F^2)$ and interprets $Q > 0$ as vorticity-dominated regions.
 - **Checks:** algebra, sign/limiting sanity check
 - **Verdict:** PASS; confidence: high; impact: moderate
 - **Assumptions/inputs:** $\|\cdot\|_F$ is the Frobenius norm.
 - **Notes:** If pure rotation: $S = 0$, $Q > 0$; if pure strain: $W = 0$, $Q < 0$. Interpretation matches the definition.
8. **✓ Force density definitions** (Sec. 2.4, p.3)
- **Claim:** Defines $\{\mathbf{F}\}_{\text{rm grav}} = -\rho \nabla \Phi$, $\{\mathbf{F}\}_{\text{rm gas}} = -\nabla p$, $\{\mathbf{F}\}_{\text{rm rad}} = -\nabla \cdot \mathbf{P}$ as the dominant force densities.
 - **Checks:** dimensional/units consistency, notation consistency

- **Verdict:** PASS; confidence: medium; impact: moderate
- **Assumptions/inputs:** Φ_R is specific potential; p_g is gas pressure; \mathbf{P}_r is radiation pressure tensor.
- **Notes:** Each term has force-per-volume units ($\rho \times$ acceleration; pressure gradient; divergence of a pressure tensor).

9. ✓ **Power density (work rate per unit volume)** (Sec. 2.4, p.3)

- **Claim:** Defines $W_i = \mathbf{F}_i \cdot \vec{v}$ and interprets sign as kinetic-energy source/sink locally.
- **Checks:** dimensional/units consistency, sign convention sanity check
- **Verdict:** PASS; confidence: high; impact: minor
- **Assumptions/inputs:** \mathbf{F}_i is force density; \vec{v} is velocity.
- **Notes:** $\mathbf{F}_i \cdot \vec{v}$ yields power per unit volume. The sign interpretation (positive when force aligns with velocity) is correct.

10. Δ **Kurtosis reference value for Gaussian** (Sec. 3.2, p.5)

- **Claim:** States that a Gaussian distribution has kurtosis **3** and uses this to interpret large kurtosis as intermittency.
- **Checks:** definition consistency, missing assumptions check
- **Verdict:** UNCERTAIN; confidence: medium; impact: minor
- **Assumptions/inputs:** Kurtosis is the 4th standardized moment (Pearson), not excess kurtosis.
- **Notes:** The comparison to **3** is correct under the Pearson (non-excess) definition, but the paper does not specify the convention used by the moment calculator; without that, the textual interpretation could be offset by **3**.

11. Δ **Undefined radial radiation flux symbol** (Sec. 3.4 and Table 3, pp.6–7)

- **Claim:** Compares ‘radial radiation flux ($F_{r,\text{rad}}$)’ between source regions and the full surface.
- **Checks:** symbol/definition consistency
- **Verdict:** UNCERTAIN; confidence: high; impact: moderate
- **Assumptions/inputs:** $F_{r,\text{rad}}$ is a component of a radiation flux vector available from the simulation or derived from radiation moments.
- **Notes:** Methods define radiation pressure tensor \mathbf{P} and radiation forcedensity $-\nabla \cdot \mathbf{P}$ is computed/sign-conventioned. This blocks a purely analytic check of consistency between the ‘radiation force’ and ‘radiation flux’ quantities discussed.} but never define a radiation flux vector or how $F_{r,\text{rad}}$

Limitations

- The paper provides few explicit derivation steps; most mathematical content is definitional, so deeper algebraic verification is limited.
- Key correctness of the Roche-lobe mask hinges on the exact Roche potential formula and reference frame; the provided text appears ambiguous/incorrect for the companion term and under-specified for the rotating-frame origin/axis.
- Implementation details (finite differencing, metric terms in spherical coordinates, connected-component segmentation) are intentionally out of scope and cannot be analytically validated here.

Numerical results audit

This section audits **numerical/empirical** consistency: reported metrics, experimental design, baseline comparisons, statistical evidence, leakage risks, and reproducibility.

16 candidate numeric checks were evaluated: 14 PASS, 1 FAIL, and 1 UNCERTAIN. The only failing check is a text claim of an “almost $20\times$ ” flux increase (computed $17.747\times$ from the reported means). Several internal consistency/inequality checks for Table 1 and ratio/threshold checks for Table 2/3 pass as stated.

Checked items

- Δ **C1** (Page 4, §3.1 (gravitational environment))
 - **Claim:** “We calculate the potential at L1 to be $\Phi_{L1} = -3.828$ (in code units)”
 - **Checks:** repeated-constant consistency
 - **Verdict:** UNCERTAIN
 - **Notes:** Cannot verify repeated-constant consistency: no document text/table corpus to scan for other Φ_{L1} occurrences.
- ✓ **C2** (Page 4, Table 1 (Density ρ row))

- **Claim:** Table 1 reports Density (ρ): Mean 8.342×10^{-2} , Std Dev 3.557×10^{-1} , Min 1.000×10^{-8} , Max 6.495×10^0 .
 - **Checks:** range and positivity checks
 - **Verdict:** PASS
 - **Notes:** All inequality checks satisfied.
3. ✓ **C3** (Page 4, Table 1 (Gas Pressure p_g row))
- **Claim:** Table 1 reports Gas Pressure (p_g): Mean 4.737×10^{-2} , Std Dev 1.444×10^{-1} , Min 1.000×10^{-12} , Max 1.213×10^0 .
 - **Checks:** range and positivity checks
 - **Verdict:** PASS
 - **Notes:** All inequality checks satisfied.
4. ✓ **C4** (Page 4, Table 1 (Radial Velocity v_r row))
- **Claim:** Table 1 reports Radial Velocity (v_r): Mean -5.345×10^{-2} , Std Dev 4.056×10^{-1} , Min -2.396×10^0 , Max 2.537×10^0 .
 - **Checks:** range checks (signed variable)
 - **Verdict:** PASS
 - **Notes:** All inequality checks satisfied. Also $|\text{mean}|/\text{std} = 0.13178$ (informational).
5. ✓ **C5** (Page 4, Table 1 (Theta Velocity v_θ row))
- **Claim:** Table 1 reports Theta Velocity (v_θ): Mean 1.140×10^{-4} , Std Dev 3.999×10^{-1} , Min -2.606×10^0 , Max 2.629×10^0 .
 - **Checks:** range checks (signed variable)
 - **Verdict:** PASS
 - **Notes:** All inequality checks satisfied.
6. ✓ **C6** (Page 4, Table 1 (Phi Velocity v_ϕ row))
- **Claim:** Table 1 reports Phi Velocity (v_ϕ): Mean -1.481×10^{-1} , Std Dev 7.258×10^{-1} , Min -3.850×10^0 , Max 1.960×10^0 .
 - **Checks:** range checks (signed variable)
 - **Verdict:** PASS
 - **Notes:** All inequality checks satisfied.
7. ✓ **C7** (Page 4, Table 1 (Radiation Energy E_r row))
- **Claim:** Table 1 reports Radiation Energy (E_r): Mean 2.268×10^{-2} , Std Dev 6.587×10^{-2} , Min 1.726×10^{-15} , Max 5.830×10^{-1} .
 - **Checks:** range and positivity checks
 - **Verdict:** PASS
 - **Notes:** All inequality checks satisfied.
8. ✗ **C8** (Page 5, §3.2 + Table 2 (mean flux ratio claim))
- **Claim:** “When we isolate a region immediately surrounding the L1 point... The mean flux is almost 20 times higher” (Global mean 0.028256 vs L1 Region mean 0.501464).
 - **Checks:** ratio recomputation
 - **Verdict:** FAIL
 - **Notes:** Computed ratio (L1/global) = 17.747168742921858; exceeds the 10% relative tolerance for “almost 20×”.
9. ✓ **C9** (Page 5, §3.2 + Table 2 (kurtosis 'over 1000'))
- **Claim:** “The PDF of j_{mag} ... is characterized by an extremely high kurtosis of over 1000.” Table 2 lists global kurtosis 1033.6.
 - **Checks:** threshold check
 - **Verdict:** PASS
 - **Notes:** Strict greater-than threshold check.
10. ✓ **C10** (Page 5, §3.2 (Gaussian kurtosis comparison) + Table 2)

- **Claim:** Text compares global kurtosis to “the value of 3 for a Gaussian distribution”; Table 2 gives global kurtosis 1033.6.
 - **Checks:** order-of-magnitude / relative comparison
 - **Verdict:** PASS
 - **Notes:** Checked $\text{global_kurtosis} > \text{gaussian_reference}$; factor over Gaussian = 344.5333333333333.
11. ✓ **C11** (Page 6, §3.4 bullet (1) + Table 3 (density ratio))
- **Claim:** “The mean density... within the source fingerprint are approximately 5 times lower than the respective surface averages” using Table 3 densities: Source 1.727×10^{-2} vs Full 8.428×10^{-2} .
 - **Checks:** ratio recomputation
 - **Verdict:** PASS
 - **Notes:** Computed full/source ratio = 4.880138969310943, consistent with “approximately $5 \times$ lower” under the stated tolerance.
12. ✓ **C12** (Page 6, §3.4 bullet (1) + Table 3 (pressure ratio))
- **Claim:** “The mean ... pressure within the source fingerprint are approximately 5 times lower than the respective surface averages” using Table 3 pressures: Source 9.657×10^{-3} vs Full 4.665×10^{-2} .
 - **Checks:** ratio recomputation
 - **Verdict:** PASS
 - **Notes:** Computed full/source ratio = 4.830692761727244, consistent with “approximately $5 \times$ lower” under the stated tolerance.
13. ✓ **C13** (Page 6, §3.4 bullet (3) + Table 3 (radiation flux ratio))
- **Claim:** “The mean outward flux from the source regions is over 6.4 times greater than the surface average.” Table 3: $F_{r,\text{rad}}$ source 2.837×10^{-3} vs full 4.428×10^{-4} .
 - **Checks:** ratio + threshold check
 - **Verdict:** PASS
 - **Notes:** Computed ratio (source/full) = 6.406955736224029, which is strictly greater than 6.4.
14. ✓ **C14** (Page 7, Table 3 (Density stats))
- **Claim:** Table 3 Density (ρ) mean and std dev for source vs full: Source mean 1.727×10^{-2} , std 5.516×10^{-2} ; Full mean 8.428×10^{-2} , std 3.637×10^{-1} .
 - **Checks:** std-dev non-negativity and plausibility bounds
 - **Verdict:** PASS
 - **Notes:** All std dev values non-negative.
15. ✓ **C15** (Page 7, Table 3 (Pressure stats))
- **Claim:** Table 3 Pressure (p_g) mean and std dev for source vs full: Source mean 9.657×10^{-3} , std 3.772×10^{-2} ; Full mean 4.665×10^{-2} , std 1.416×10^{-1} .
 - **Checks:** std-dev non-negativity
 - **Verdict:** PASS
 - **Notes:** All std dev values non-negative.
16. ✓ **C16** (Page 7, Table 3 (Velocity mean within implied min/max not provided))
- **Claim:** Table 3 lists mean radial velocity: source -1.481×10^{-2} vs full -5.987×10^{-2} ; claim: “mean radial velocity in the source regions is significantly less negative.”
 - **Checks:** signed comparison
 - **Verdict:** PASS
 - **Notes:** Checked $v_{r,\text{mean},\text{source}} > v_{r,\text{mean},\text{full}}$ (less negative); difference = 0.04506.

Limitations

- Only parsed text was available; no access to underlying simulation data arrays, so any claim requiring recomputation from fields (Q-criterion segmentation, force balance maps, PDFs) cannot be verified.
- No plot digitization was performed; numeric checks are limited to explicit table values and explicit numeric statements in the text.
- Some qualitative phrases (e.g., 'significantly', 'conservative threshold', 'objects of significant volume') lack quantitative criteria, limiting what can be asserted with automated fast checks.

- Repeated-constant consistency checks (e.g., verifying whether $\Phi_{L1} = -3.828$ is used consistently throughout) could not be completed because the checking routine could not scan a full document corpus for other occurrences.