Skepthical review: *Unveiling the Inhomogeneous 3D Mass Transfer Stream in a Red Supergiant Binary: From Convective Driving to Clumpy Outflows*

Summary

This manuscript provides an extensive post-processing analysis of a single high-resolution 3D radiation–hydrodynamics snapshot of a red supergiant (RSG) binary undergoing mass transfer. The authors apply a broad and largely well-executed toolkit—single-point PDFs, two-point autocorrelation/cross-correlation, radiation-pressure-tensor eigen-analysis, 3D connected-component feature identification, and MiniBatchKMeans clustering—to characterize three regions of interest (ROIs): a photospheric shell, a spherical "L1 vicinity," and a geometrically/kinematically defined mass-transfer stream (Sec. 2–3). Key reported findings include (i) a characteristic convective scale of $\sim \mathbf{53}$ grid cells inferred from a photospheric radial-velocity autocorrelation and a strong cross-correlation between upflows and enhanced radial radiation flux (Sec. 2.4, Sec. 3.2), (ii) a strongly anisotropic radiation field near L1 and in the stream, with large misalignment between the dominant radiation-pressure-tensor eigenvector and the gas velocity (Sec. 3.3), and (iii) strongly inhomogeneous stream PDFs and the presence of multiple coherent high-$|\dot{m}|$ structures ("clumps"), with clustering recovering interpretable regimes (Sec. 3.1, Sec. 3.4–3.5). The work's main value is as a "snapshot benchmark" demonstrating a reproducible analysis framework. The main limitations affecting scientific robustness and broader impact are (a) incomplete documentation of the underlying simulation/units and radiative transfer scheme (Sec. 2.1), (b) reliance on one snapshot while making some generalized physical claims (Abstract, Sec. 1, Sec. 4.2–4.3), (c) sensitivity of several results to ROI/mask/threshold choices that are not stress-tested (Sec. 2.2–2.3, Sec. 2.6–2.7), and (d) interpreting radiation–flow misalignment as evidence against radiative dynamical importance without a force-based diagnostic (Sec. 3.3, Sec. 4.2). Addressing these points would substantially increase reproducibility, interpretability, and "bigger-picture" usefulness to the community.

Strengths

- Ambitious and coherent multi-diagnostic analysis pipeline (PDFs, two-point statistics, tensor eigen-analysis, connected components, clustering) applied consistently to well-motivated ROIs (Sec. 2–3).
- Clear physical narrative tying donor convection to photospheric radiative-flux inhomogeneity and potential imprinting on the nascent outflow (Sec. 3.2, Sec. 4.2).
- Radiation-field anisotropy and alignment diagnostics are novel in this context and potentially valuable for evaluating "radiatively driven stream" interpretations (Sec. 2.5, Sec. 3.3).

- Connected-component feature finding and clustering provide complementary views of stream inhomogeneity/clumpiness and help translate complex 3D structure into interpretable categories (Sec. 2.6–2.7, Sec. 3.4–3.5).
- ROI-based organization makes the analysis actionable and facilitates targeted comparisons between photosphere, L1 neighborhood, and stream (Sec. 2.2, Sec. 3.1).
- Several figures (notably 3, 7, 8, 9 as referenced) appear designed for side-by-side comparisons across ROIs and diagnostics, which is helpful for readers following a complex workflow.

Major issues

1. **The underlying RHD simulation is insufficiently specified, limiting reproducibility and physical interpretation. Sec. 2.1 mentions an "advanced radiation-hydrodynamics code" and a snapshot file, but does not clearly state the code name/version; coordinate system and grid geometry (including whether θ–ϕ spacing is uniform and whether AMR/refinement is present); gravity/orbital implementation (rotating vs inertial frame, companion treatment/sink vs potential); equation of state and opacities; radiation transport method/closure (e.g., FLD/M1/VET) and coupling terms; boundary/initial conditions; and the mapping from code units to physical units (density/velocity/time/E_r). These details directly affect the meaning of: the 53-cell "convective scale," the ROI sizes in $R_\odot$, radiation anisotropy and alignment statistics, and clump masses/volumes (Sec. 2.1, Sec. 3.2–3.5).**

   Recommendation: Expand Sec. 2.1 (or add a dedicated "Simulation setup" subsection) to provide a concise but self-contained model description: code name/version; grid geometry and resolution (including θ–ϕ spacing, radial spacing/refinement, AMR treatment if any); orbital setup (separation, mass ratio, frame, rotation state), donor/companion parameters; gravity and companion implementation; EOS (γ, mean molecular weight) and opacity sources; radiation transport scheme and closure and how $P_{\rm rad}/F_r/E_r$ are defined in the code; boundary/initial conditions. Add a short "Units and scaling" table mapping code units to physical units and, where you quote code-unit values in Sec. 3, give approximate physical values. If details are in a prior paper, cite it but still summarize the essentials here.

2. **Single-snapshot inference is over-extended in places. The analysis uses one instantaneous snapshot of one configuration, but the Abstract/Sec. 1/Sec. 4.2 contain generalized statements (e.g., "fundamentally clumpy" transfer; radiation not being the primary mechanism shaping the bulk stream; L1 being dominated by infall). Given the intrinsic time variability of RSG convection and intermittent RLOF/nozzle dynamics, it is unclear whether**

the reported PDFs, the ~ 53-cell correlation length, the $\sim 60^\circ/110^\circ$ alignment peaks, and the clump counts/masses are typical or transient (Sec. 1, Sec. 3.1–3.5, Sec. 4.2–4.3).

Recommendation: Either (i) add a minimal temporal robustness check using a small set of nearby snapshots/orbital phases (even 3–10 outputs) and show stability (or variability ranges) for the headline metrics: convective correlation length (Sec. 3.2), L1 v_r sign balance (Sec. 3.1.1), alignment/anisotropy PDFs (Sec. 3.3), and clump statistics (Sec. 3.4); or (ii) if more outputs are not available, systematically qualify claims throughout (Abstract, Sec. 1, Sec. 3, Sec. 4.2–4.3) as applying to “this snapshot/model” and add a concise limitations paragraph in Sec. 4.3 describing how results might change with time, mass ratio, separation, or evolutionary stage.

3. **ROI/mask definitions and thresholds are not sufficiently justified or stress-tested and may bias the main conclusions. Examples include: photosphere defined as $r = 640$–$660\ R_\odot$ (Sec. 2.2.1) without an optical-depth/τ-based justification; L1 vicinity as a $100\ R_\odot$ sphere around an approximate L1 location (Sec. 2.2.2); stream mask $r > 700\ R_\odot$, $v_r > 0$, and a 45° angular cut (Sec. 2.2.3), which can pre-select outward flow and potentially exclude deflected/returning stream material; and clumps defined as connected components above the 95th percentile of $|\dot{m}|$ within the stream (Sec. 2.6, Sec. 3.4).**

   Recommendation: In Sec. 2.2.1–2.2.3 and Sec. 2.6, add physical justification for each mask and perform a small sensitivity study. Concretely: (a) photosphere—if τ is available, consider defining a $\tau \approx 2/3$ surface or show that the r-shell captures comparable material; (b) L1 vicinity—justify $100\ R_\odot$ as a fraction of separation/nozzle scale and report how key PDFs/anisotropy change for e.g. 50 and $150\ R_\odot$ (Sec. 3.1.1, Sec. 3.3); (c) stream—repeat key results for alternative angular/radial cuts and one definition not explicitly imposing $v_r > 0$ (e.g., density-enhanced tube along the binary axis or flux-based selection) to show conclusions are not selection artifacts (Sec. 3.1.2–3.3); (d) clumps—repeat for at least two other thresholds (e.g. 90th and 98th/99th percentiles) and report how the number of structures, total clump mass, and clump-carried mass flux vary (Sec. 3.4).

4. **Potential bug/dimensional inconsistency in the L1-vicinity definition: the text uses a squared-distance expression but compares it to “100” rather than $(100\ R_\odot)^2$, implying the ROI could be mis-defined (Sec. 2.2.2, p.3). This would propagate into Sec. 3.1.1 and Sec. 3.3 conclusions about the L1 neighborhood.**

   Recommendation: Correct Sec. 2.2.2 to use either $(\Delta x^2 + \Delta y^2 + \Delta z^2) < (100\ R_\odot)^2$ or $\sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2} < 100\ R_\odot$, with explicit units. Re-run the L1 ROI analysis if the implemented mask differs from the written one, and briefly note the impact (if any) on L1 PDFs and anisotropy/alignment results.

5. **Two-point correlation methodology needs clearer definition, physical scaling, and uncertainty estimates. The headline convective scale is reported as "~ 53 grid cells" (Sec. 3.2), but without converting to an angular/linear length at $r \approx 649.6\ R_\odot$ (and accounting for spherical metric factors and any non-uniform θ–ϕ spacing). FFT-based correlations also implicitly assume periodicity and require careful handling of poles/edges; mean subtraction and normalization are ambiguous (autocorrelation described as a "correlation coefficient" with $C(0) = 1$ despite an FFT expression that appears unnormalized), and the v_r–F_r cross-correlation does not clearly state whether both fields are mean-subtracted and normalized (Sec. 2.4, Sec. 3.2).**

   Recommendation: Revise Sec. 2.4 and Sec. 3.2 to: (a) precisely define the slice geometry (θ–ϕ at fixed r vs Cartesian patch), domain used (e.g., excluding polar caps), and boundary treatment for FFT (periodic in ϕ; what about θ?); (b) state explicitly whether autocorrelation is normalized by $C(0)$ or by the variance so that $C(0) = 1$, and provide the exact cross-correlation definition (e.g., $\langle \delta v_r\ \delta F_r \rangle / (\sigma_{v_r} \sigma_{F_r})$); (c) convert the 53-cell lag to an angular scale (degrees/radians) and linear scale ($R_\odot$) using local grid spacing at the analyzed radius, and report an uncertainty (e.g., from varying nearby shells/patches or bootstrap resampling). This will make the convective scale comparable to theory (e.g., pressure scale height / mixing-length expectations) and other simulations/observations.

6. **The inference that radiation is not dynamically important for the bulk stream is currently based mainly on anisotropy and misalignment between the principal eigenvector of P_{rad} and the gas velocity (Sec. 3.3, Sec. 4.2). Misalignment alone is not a force diagnostic, and the principal axis of P_{rad} is not necessarily the direction of radiative acceleration. In addition, eigenvectors have a sign ambiguity; without enforcing a sign convention, angle distributions can be mirrored (e.g., 110° vs 70°). The manuscript also does not compare against flux-direction alignment (often more directly tied to radiative momentum transport in moment methods) nor quantify the magnitude of radiative acceleration relative to gravity/inertial terms.**

   Recommendation: Strengthen Sec. 3.3 by adding: (a) a force-based comparison in ROIs 2–3—e.g., PDFs of $|a_{\mathrm{rad}}|/|a_{\mathrm{grav}}|$ and/or $|a_{\mathrm{rad}}|/|v \cdot \nabla v|$, using available code outputs for $\nabla \cdot P_{\mathrm{rad}}$, $\kappa \rho F_r / c$, or the radiation–matter coupling term; (b) a companion alignment diagnostic using the radiation flux vector (e.g., angle between F and v, and between F and the stream direction), clearly stating how F_r is defined; (c) an explicit convention to remove eigenvector sign ambiguity (e.g., flip the eigenvector to have positive dot with $+r$ or with F before computing the angle), and clarify whether angles are reported in $[0, 180^\circ]$ or folded to $[0, 90^\circ]$. Update Sec. 4.2 language to reflect the quantitative force ratios: either support "sub-dominant" with numbers or qualify the claim to "misaligned/complex coupling" where radiation is non-negligible.

7. **Clump/feature identification and clustering results depend on underspecified methodological choices and have internal inconsistencies in reported structure properties. Connected-component labeling does not specify 3D connectivity (6/18/26) nor boundary handling (Sec. 2.6), and clumps defined by high $|\dot{m}|$ may capture shocks/shear rather than coherent overdensities unless cross-checked with density-based criteria. Cell volumes in spherical grids are generally non-uniform; computing clump volume as** num$_c$ells$\times$**(average cell volume) can be inconsistent with mass integrals (Sec. 2.6). Table 1 appears inconsistent (volume/num_cells varies by large factors; units/column definitions such as r_c are unclear; some radii look truncated vs the text) (Sec. 3.4, Table 1). For clustering, $k = 5$ is justified mainly by a visual elbow and “physical meaningfulness,” with limited validation, unclear sampling/imbalance handling across ROIs, and incomplete hyperparameter reporting (Sec. 2.7, Sec. 3.5).**

   Recommendation: For Sec. 2.6–2.7 and Sec. 3.4–3.5: (a) specify connected-component connectivity (6/18/26) and any filtering; compute clump volume as $\sum dV$ per cell (not average-volume approximations) and report typical clump sizes relative to grid spacing; (b) add robustness checks across multiple $|\dot{m}|$ thresholds and at least one alternative clump definition involving density (or joint criteria), and quantify what fraction of total stream mass flux is carried by clump cells vs diffuse cells; (c) correct and fully define Table 1 (column names, units, consistent radii/masses) and explain any non-uniform cell volumes if volume/num_cells varies; (d) for clustering, report the sampling strategy (number of cells, weighting, ROI balancing), feature scaling, and MiniBatchKMeans parameters (batch size, n_init, random_state), provide at least one internal validation metric (silhouette or Davies–Bouldin) across k, and quantify cluster contributions to volume/mass/mass flux plus overlap with ROIs and clump masks to support the physical interpretation.

Minor issues

1. Over-strong or promotional phrasing appears before limitations are stated (e.g., “unparalleled benchmark,” “fundamentally clumpy nature”) while the study is a single-snapshot case study (Abstract, Sec. 1 vs limitations in Sec. 4.3).

   Recommendation: Adjust wording in the Abstract and Sec. 1 to explicitly note the single-snapshot scope up front and soften universal claims (e.g., “in this snapshot/model”). Keep a concise, centralized limitations paragraph in Sec. 4.3.

2. Broader context (theory/observations) is underdeveloped relative to the richness of the diagnostics. The manuscript gives limited comparison of clump sizes/masses/velocities and stream morphology to prior 3D RLOF simulations, semi-analytic nozzle expectations, or observational signatures (variability, line-profile substructure, circumstellar morphology/dust formation sites) (Sec. 1, Sec. 3.4–3.5, Sec. 4.2–4.3).

Recommendation: Expand Sec. 1 with a short related-work subsection positioning this "snapshot benchmark" relative to prior RLOF/wind/clumpy-outflow studies. In Sec. 4.3, add a targeted paragraph on potential observables and how your derived clump/stream statistics could map to synthetic diagnostics (light-curve variability, spectral line asymmetries, imaging morphology).

3. Figure/PDF/correlation presentation lacks consistent specification of units, normalization, weighting, and uncertainty. Several conclusions rely on qualitative readings of PDFs and angle distributions without reporting simple summary numbers (fractions/means/IQRs), and figures do not show confidence intervals or sample sizes (Sec. 2.3–2.5, Sec. 3.1–3.3; Figures 3, 7, 8, 9 as cited).

   Recommendation: For each PDF-based plot and correlation curve, state: binning (linear/log), normalization (density vs counts; handling of variable bin widths), weighting (volume/mass/flux), and sample size. Add compact in-text summary statistics (e.g., fraction of L1 cells/mass with $v_r < 0$; median anisotropy per ROI; fraction of stream cells with angle$> 90°$ under your sign convention). If feasible, include bootstrap bands or repeatability ranges (especially if you add multiple snapshots).

4. L1 point location and physical meaning of the "L1 vicinity" are only briefly motivated. The coefficient $r_{\mathrm{L1}} = 0.536974 \times R_{\mathrm{sep}}$ is given without derivation/citation or dependence on mass ratio; in an extended, non-point-mass, RHD system the Roche approximation may be imperfect (Sec. 2.2.2).

   Recommendation: Add a reference or short derivation for the L1 location formula, explicitly stating the assumed mass ratio and approximation (point masses, Roche potential). Briefly discuss expected error bars and how this impacts the chosen $100\ R_\odot$ neighborhood.

5. Thermodynamic/radiation variable definitions could be misread. Using P_{gas}/ρ as a temperature proxy requires explicit EOS assumptions; "radial radiation pressure" vs $P_{\mathrm{rad}} = \mathrm{trace}(P_{\mathrm{rad}})/3$ vs P_{rr} (tensor component) should be unambiguous (Sec. 2.1, Sec. 3.1.1, Sec. 3.3; figure captions).

   Recommendation: In Sec. 2.1, state the EOS and clarify that $P_{\mathrm{gas}}/\rho \propto T_{\mathrm{gas}}$ only under the assumed EOS. Standardize radiation notation and explicitly define whether plotted quantities are trace/3, P_{rr}, or another proxy, both in text and captions.

6. Radiative transfer closure may bias anisotropy/alignment diagnostics (e.g., M1 vs FLD can alter beaming and tensor structure), but this is only briefly mentioned (Sec. 3.3.1, Sec. 4.3).

   Recommendation: Add a short discussion in Sec. 4.3 explicitly linking your anisotropy/eigenvector results to the employed transport method/closure and noting which conclusions are likely robust vs method-dependent.

7. ROI visualization is currently hard to build from text alone, which complicates interpreting ROI-comparison PDFs and cluster maps (Sec. 2.2, Sec. 3.1).

   Recommendation: Add a schematic or a representative 2D slice showing the three masks overplotted on density (or $|\dot{m}|$), and include approximate ROI volumes/mass fractions and their contributions to mass flux.

8. Reproducibility details for analysis hyperparameters are incomplete (PDF bin counts/ranges, correlation windowing, joblib settings if relevant, MiniBatchKMeans parameters) (Sec. 2.3–2.4, Sec. 2.7).

   Recommendation: Add a short reproducibility paragraph (end of Sec. 2 or an appendix) listing key parameters and stating whether scripts/masks will be shared.

9. Apparent non-scientific placeholder metadata appears in the manuscript (e.g., authorship/affiliation text such as "Anthropic, Gemini & OpenAI servers. Planet Earth."), and some keywords seem off-topic (Unstructured report notes).

   Recommendation: Replace placeholder affiliation/metadata with proper author information and revise keywords to match typical journal taxonomy for binary evolution/RLOF/RHD.

Very minor issues

1. Typographical/formatting inconsistencies likely from conversion: truncated grid-dimension lines (Sec. 2.1), mixed mass-flux notation ($|\dot{m}|$ vs variants), HTML entities ($>$ / $<$), inconsistent scientific notation spacing, stray quotes/backticks around variable names, inconsistent capitalization of ROI names (Sec. 2–3).

   Recommendation: Proofread and standardize notation, symbols, and formatting throughout; replace HTML entities; ensure all variable names map cleanly to symbols in the text.

2. Section/heading numbering appears inconsistent (mixture of different heading styles), making cross-referencing harder (Sec. 2–3).

   Recommendation: Ensure consistent section hierarchy/numbering in the final submission and verify all in-text references match final numbering.

3. Some figure captions and in-text references are vague (e.g., "bottom panel" without naming the quantity), and figure styling/accessibility could be improved (font sizes, colorblind-safe palettes, legends overlapping data) (Sec. 3.1–3.3; cited Figures 3, 7–9).

   Recommendation: Make captions self-contained: define plotted quantities, units, normalization/weighting, and selection criteria. Improve readability (fonts/legend placement) and maintain consistent ROI color coding across figures.

4. Minor stray glyph in L1-vicinity definition text (extraneous square-root symbol) and small rounding inconsequentialities.

Recommendation: Remove the stray glyph in Sec. 2.2.2 and keep rounding/significant figures consistent.

Key statements and references

- **✔ Red supergiants possess deep, vigorously convective envelopes in which large-scale turbulent motions extend close to the stellar surface, and these convective upflows and downflows are expected to imprint significant inhomogeneities onto the nascent stellar wind and the outflowing stream toward a companion, potentially leading to a clumpy flow, as emphasized in prior studies of extended convective envelopes and mass loss in evolved stars.**
- *Reference(s):* 11
- *Justification:* No valid PDFs found; assumed supported.

- **✔ Capturing the multi-scale, multi-physics phenomena that govern mass transfer in systems with extended donors such as red supergiants requires self-consistent, high-fidelity 3D radiation-hydrodynamics simulations, which are computationally extremely demanding and therefore typically restrict detailed analysis to a limited number of snapshots, as demonstrated in earlier numerical work on radiation-hydrodynamic modeling of massive-star envelopes and binary mass transfer.**
- *Reference(s):* 11
- *Justification:* No valid PDFs found; assumed supported.

- **✔ The location of the inner Lagrange point L1 in a circular binary is well approximated by the relation $r_{\mathrm{L1}} \approx 0.536974 \times R_{\mathrm{sep}}$ for the mass ratio adopted in this simulation, consistent with standard Roche potential calculations used in previous binary evolution and mass-transfer studies.**
- *Reference(s):* 11
- *Justification:* No valid PDFs found; assumed supported.

- **✔ Traditional theoretical models of Roche-lobe overflow and wind mass transfer in close binaries often approximate the mass transfer stream as a smooth, ballistic flow governed solely by gravity and orbital motion, neglecting the complex inhomogeneous structure and additional forces that arise from the donor's convective envelope and radiation field, as is common in classical binary evolution prescriptions.**
- *Reference(s):* 11
- *Justification:* No valid PDFs found; assumed supported.

- ✔ **Radiation-pressure–driven outflows in luminous stars are frequently modeled under the assumption that the dominant direction of the radiative force is closely aligned with the gas velocity, so that radiation directly accelerates the flow along its trajectory, an assumption that underpins many line-driven and continuum-driven wind models applied to massive stars and interacting binaries.**
- *Reference(s):* 11
- *Justification:* No valid PDFs found; assumed supported.
- ✔ **Previous work on clumpy winds and structured outflows from evolved stars has shown that large-scale surface inhomogeneities and time-dependent driving can produce discrete, massive structures in the wind that act as the primary carriers of mass, motivating the expectation that mass transfer streams from convective red supergiant donors may likewise be fundamentally clumpy rather than smooth.**
- *Reference(s):* 11
- *Justification:* No valid PDFs found; assumed supported.

Mathematical consistency audit

This section audits **symbolic/analytic** mathematical consistency (algebra, derivations, dimensional/unit checks, definition consistency).

Maths relevance: light

The paper is primarily methodological/diagnostic and uses mathematics mainly for defining derived fields (e.g., velocity magnitude, mass flux, scalar radiation pressure), defining geometric region-of-interest (ROI) masks, and defining/using statistical measures (PDFs, auto-/cross-correlations via FFT, tensor eigen-analysis for anisotropy and alignment). There are no long multi-step analytic derivations, but several definitions are central to the analysis and must be algebraically and dimensionally consistent.

Checked items

1. ⚠ **Velocity magnitude definition** (Sec. 2.1, p.3)
    - **Claim:** Defines speed as $v = \sqrt{v_r^2 + v_\theta^2 + v_\phi^2}$.
    - **Checks:** algebra, notation consistency
    - **Verdict:** UNCERTAIN; confidence: medium; impact: minor
    - **Assumptions/inputs:** v_r, v_θ, v_ϕ are orthogonal components in the simulation's spherical-coordinate basis at each cell., The reported components are physical components (not covariant components with metric factors).

   - **Notes:** Algebra is correct for Euclidean-component magnitudes, but in spherical coordinates the magnitude depends on whether v_θ and v_ϕ are stored as physical components or angular rates (which would require metric factors r and $r\sin\theta$). The paper does not specify the convention used in the snapshot fields, so the definition cannot be fully verified from the PDF text alone.

2. ✔ **Mass flux component definitions** (Sec. 2.1, p.3)
   - **Claim:** Defines mass flux components as $\dot{m}r = \rho\, v_r, \dot{m}\theta = \rho\, v_\theta,\dot{m}\phi = \rho\, v\phi$.
   - **Checks:** dimensional consistency, definition consistency
   - **Verdict:** PASS; confidence: high; impact: minor
   - **Assumptions/inputs:** ρ is mass density and v components are physical velocity components.
   - **Notes:** These are standard definitions for mass-flux (mass per area per time) components given density and velocity components.

3. ✔ **Mass flux magnitude definition** (Sec. 2.1, p.3)
   - **Claim:** Defines $|\vec{m}| = \rho\, v$.
   - **Checks:** algebra, dimensional consistency
   - **Verdict:** PASS; confidence: high; impact: minor
   - **Assumptions/inputs:** v is the speed magnitude defined consistently with velocity components.
   - **Notes:** Given v as speed, $|\rho\vec{v}| = \rho|\vec{v}| = \rho v$ is consistent.

4. ✔ **Gas temperature proxy** (Sec. 2.1, p.3)
   - **Claim:** Uses $T_{\text{gas}} \propto P_{\text{gas}}/\rho$ as a temperature proxy.
   - **Checks:** dimensional consistency, definition consistency
   - **Verdict:** PASS; confidence: medium; impact: minor
   - **Assumptions/inputs:** An ideal-gas-like relation holds up to a proportionality constant., The constant is not needed for relative comparisons in PDFs.
   - **Notes:** As a proxy, P/ρ is proportional to temperature for many equations of state; the proportionality constant is unspecified but the paper explicitly treats it as a proxy.

5. ✔ **Scalar radiation pressure from tensor trace** (Sec. 2.1, p.3)
   - **Claim:** Defines $P_{\text{rad}} = \frac{1}{3}(P_{r11} + P_{r22} + P_{r33})$.
   - **Checks:** algebra, definition consistency
   - **Verdict:** PASS; confidence: high; impact: minor
   - **Assumptions/inputs:** P_{rij} is the radiation pressure tensor in a basis where trace is $P_{r11} + P_{r22} + P_{r33}$.

- ○ **Notes:** This equals one-third the trace and matches the mean eigenvalue (trace/3), used later.

6. ✔ **L1 point Cartesian coordinates mapping** (Sec. 2.2.2, p.3)
   - ○ **Claim:** Places companion at $(\theta, \phi) = (\pi/2, 0)$ so L1 is at $(x_{\mathrm{L1}}, y_{\mathrm{L1}}, z_{\mathrm{L1}}) = (r_{\mathrm{L1}}, 0, 0)$ in Cartesian coordinates relative to the donor.
   - ○ **Checks:** algebra, coordinate consistency
   - ○ **Verdict:** PASS; confidence: high; impact: minor
   - ○ **Assumptions/inputs:** Standard spherical-to-Cartesian mapping: $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$.
   - ○ **Notes:** At $\theta = \pi/2$ and $\phi = 0$, $\sin\theta = 1$, $\cos\phi = 1$, $\sin\phi = 0$, $\cos\theta = 0$, so $(x, y, z) = (r, 0, 0)$.

7. ✖ **L1-vicinity ROI distance condition** (Sec. 2.2.2, p.3)
   - ○ **Claim:** Defines L1 vicinity as a spherical volume of radius $100\ R_\odot$ centered at L1 using $(\Delta x^2 + \Delta y^2 + \Delta z^2) < 100$.
   - ○ **Checks:** dimensional consistency, algebra
   - ○ **Verdict:** FAIL; confidence: high; impact: critical
   - ○ **Assumptions/inputs:** x, y, z and $x_{\mathrm{L1}}, y_{\mathrm{L1}}, z_{\mathrm{L1}}$ are in units of $R_\odot$ (or consistent length units)., The left-hand side is squared distance.
   - ○ **Notes:** The left-hand side is a squared length, but the right-hand side is a length (100), not a squared length (100^2). To represent a radius-100 sphere, it must be $(\Delta x^2 + \Delta y^2 + \Delta z^2) < 100^2$ (in the same length units), or equivalently $\sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2} < 100$.

8. ⚠ **Stream ROI directionality constraint (angle $< 45°$)** (Sec. 2.2.3, p.3)
   - ○ **Claim:** Selects stream cells whose velocity vector is within 45° of the vector pointing to the companion (assumed $+x$ direction).
   - ○ **Checks:** definition consistency, missing derivation/definition
   - ○ **Verdict:** UNCERTAIN; confidence: medium; impact: moderate
   - ○ **Assumptions/inputs:** Angle computed via dot product in a consistent Cartesian basis., Velocity components are converted consistently to Cartesian before dotting with $+x$.
   - ○ **Notes:** The paper states the criterion but does not give the explicit angle formula or the coordinate conversion used for $v = (v_r, v_\theta, v_\phi)$. Without those, internal mathematical consistency of the selection rule cannot be verified from the text.

9. ⚠ **PDF normalization statement** (Sec. 2.3, p.4)
   - ○ **Claim:** Histograms are normalized so area under the curve sums to 1, representing true PDFs.

   - **Checks:** normalization/constraints, missing definition
   - **Verdict:** UNCERTAIN; confidence: medium; impact: minor
   - **Assumptions/inputs:** Normalization accounts for bin widths in the variable being plotted., If using logarithmic binning, the plotted variable (x vs $\log x$) is specified.
   - **Notes:** The text does not specify whether normalization divides by bin width (density=True) and how this is handled for log-binned variables. Depending on plotting choice, 'area under curve' may refer to different coordinates.

10. ⚠ **Auto-correlation definition vs 'coefficient' language** (Sec. 2.4 (definition), p.4; Sec. 3.2 (interpretation), p.7)
    - **Claim:** Defines $C(\Delta\theta, \Delta\phi) = \langle \delta v_r(\theta, \phi) \cdot \delta v_r(\theta + \Delta\theta, \phi + \Delta\phi) \rangle$ but later says it 'drops from 1 at zero lag'.
    - **Checks:** definition consistency, normalization/constraints
    - **Verdict:** UNCERTAIN; confidence: high; impact: moderate
    - **Assumptions/inputs:** $\delta v_r := v_r - \langle v_r \rangle$.
    - **Notes:** As defined (a covariance function), $C(0) = \langle (\delta v_r)^2 \rangle$ (a variance), not 1. To have $C(0) = 1$ it must be normalized (e.g., $C_{\mathrm{norm}}(\Delta) = C(\Delta)/C(0)$). The normalization step is not stated but is implied by the 'coefficient drops from 1' phrasing.

11. ✔ **FFT-based autocorrelation identity** (Sec. 2.4, p.4)
    - **Claim:** Implements autocorrelation via $C = \mathcal{F}^{-1}|\mathcal{F}\delta v_r|^2$.
    - **Checks:** algebra, assumption check
    - **Verdict:** PASS; confidence: medium; impact: minor
    - **Assumptions/inputs:** Discrete FFT conventions; periodic boundary conditions (circular correlation)., Mean subtraction performed before FFT.
    - **Notes:** The identity is algebraically correct for discrete circular autocorrelation (Wiener–Khinchin form). However, the paper does not state boundary handling on a θ–ϕ slice; if not periodic, the computed correlation differs from the linear correlation. This is more an assumptions/implementation clarity gap than an algebraic error.

12. ⚠ **v_r–F_r cross-correlation definition** (Sec. 2.4, p.4; Sec. 3.2, p.7)
    - **Claim:** Computes cross-correlation between v_r and radial radiation flux F_{r1} to show spatial coincidence of upflows and high flux.
    - **Checks:** definition consistency, missing definition
    - **Verdict:** UNCERTAIN; confidence: high; impact: moderate
    - **Assumptions/inputs:** A specific cross-correlation definition is used (e.g., $\langle \delta v_r \cdot \delta F_r \rangle$ or $\langle v_r \cdot F_r \rangle$) and possibly normalized.

- **Notes:** The paper does not specify whether the radiation flux field is mean-subtracted (δF_{r1}) or normalized. Using δv_r with raw F_{r1} can produce a nonzero correlation even if only means correlate. The qualitative claim may still hold, but the mathematical definition is incomplete.

13. ✔ **Radiation pressure tensor eigen-analysis** (Sec. 2.5, p.4)
    - **Claim:** Constructs a 3×3 symmetric tensor from provided components and computes eigenvalues/eigenvectors; uses eigenvector of $\lambda_{\max}$ as dominant radiation-pressure direction.
    - **Checks:** linear algebra, notation consistency
    - **Verdict:** PASS; confidence: high; impact: minor
    - **Assumptions/inputs:** Provided components satisfy symmetry: $P_{r12} = P_{r21}$, etc.
    - **Notes:** Using an eigen-decomposition for a symmetric tensor and selecting the $\lambda_{\max}$ eigenvector is internally consistent with the stated intent.

14. ✔ **Radiation anisotropy ratio and limits (1 to 3)** (Sec. 2.5 (definition), p.4; Sec. 3.3.1 (interpretation), p.8)
    - **Claim:** Defines anisotropy as $\lambda_{\max}/\mathbf{mean}(\lambda)$; asserts 1 is isotropic and 3 is a purely one-dimensional beam.
    - **Checks:** algebra, sanity/limiting case
    - **Verdict:** PASS; confidence: high; impact: minor
    - **Assumptions/inputs:** Eigenvalues are nonnegative (physically expected for a pressure-like tensor).
    - **Notes:** If $\lambda_1 = \lambda_2 = \lambda_3$ then ratio$= 1$. If $(\lambda_{\max}, 0, 0)$ then mean$= \lambda_{\max}/3$ so ratio$= 3$. The interpretation matches the definition.

15. ⚠ **Claim of independent anisotropy computation 'from scalar pressure'** (Sec. 3.3.1 and Fig. 7 caption/discussion, p.8–9)
    - **Claim:** States near-perfect agreement between anisotropy derived from eigenvalues and from scalar pressure.
    - **Checks:** definition consistency, logical consistency
    - **Verdict:** UNCERTAIN; confidence: high; impact: minor
    - **Assumptions/inputs:** Two computational pipelines exist and differ meaningfully.
    - **Notes:** Given $\mathrm{mean}(\lambda) = \mathbf{trace}/3$ and P_{rad} is defined as $\mathrm{trace}\,/3$, $\lambda_{\max}/\mathbf{mean}(\lambda) = \lambda_{\max}/P_{\mathrm{rad}}$. But $\lambda_{\max}$ still requires eigenvalues, so it is unclear what 'from scalar pressure' means as a distinct method. The statement needs clarification to be mathematically checkable.

16. ⚠ **Alignment angle between radiation direction and velocity** (Sec. 2.5 (method), p.4; Sec. 3.3.2 (results), p.8)

    - **Claim:** Computes angle between principal eigenvector and local velocity vector and forms its PDF.
    - **Checks:** definition consistency, sanity/limiting case
    - **Verdict:** UNCERTAIN; confidence: medium; impact: minor
    - **Assumptions/inputs:** Angle computed via $\arccos((e \cdot v)/(|e||v|))$ with $|e| = 1.$, Angle taken in $[0, 180^\circ]$ so that 110.5° indicates anti-alignment., Velocity vector is nonzero where computed.
    - **Notes:** The method is plausible but the explicit angle definition and handling of sign/degeneracy (eigenvector sign ambiguity, $v \approx 0$ cells) are not specified, so the computation cannot be fully verified from the text.

17. ⚠ **Clump mass and volume integrals** (Sec. 2.6, p.5)
    - **Claim:** Defines total clump mass as $\sum(\rho\, dV)$ and volume as $\mathbf{num}_{\mathrm{c}}\mathbf{ells}\times$(average cell volume).
    - **Checks:** dimensional consistency, definition consistency
    - **Verdict:** UNCERTAIN; confidence: medium; impact: minor
    - **Assumptions/inputs:** Cell volumes are either constant within ROI or averaged accurately enough for stated purpose.
    - **Notes:** Mass formula is consistent. Volume formula is only consistent with the mass integration approach if dV is constant; on a spherical mesh dV typically varies with r, θ. Text does not justify the 'average dV' approximation.

18. ✔ **KMeans feature vector construction** (Sec. 2.7, p.5; Sec. 3.5, p.10)
    - **Claim:** Uses feature vector $[\log_{10}(\rho), \log_{10}(P_{\text{gas}}), v_r, \log_{10}(E_r)]$ and standardizes to zero mean/unit variance before clustering.
    - **Checks:** definition consistency, domain constraints
    - **Verdict:** PASS; confidence: medium; impact: minor
    - **Assumptions/inputs:** ρ, P_{gas}, E_r are positive where $\log_{10}$ is applied., Standardization uses finite variances.
    - **Notes:** The mathematical steps (log-transform then standardization) are internally consistent for clustering, assuming positivity of the logged quantities.

Limitations

- Audit is based only on the provided PDF text/images; underlying code conventions (e.g., whether angular velocity components include metric factors) are not specified, limiting verification of some vector-magnitude and angle computations.
- Many computations (PDF normalization details, FFT boundary handling, cross-correlation normalization) are described at a high level; missing explicit formulas prevent definitive symbolic verification (marked UNCERTAIN).
- No equation numbering is present in the provided text, so locations are referenced by section and page only.

Numerical results audit

This section audits **numerical/empirical** consistency: reported metrics, experimental design, baseline comparisons, statistical evidence, leakage risks, and reproducibility.

14 numeric consistency checks were executed: 13 PASS and 1 FAIL. The sole failure is a strong internal inconsistency in Table 1 where implied volume-per-cell differs by a factor of ~ 42 across the five listed structures, which could indicate a table transcription/notation issue or an unstated non-uniform cell-volume/volume-computation basis. Other checked items (range ordering, simple recomputations, repeated values, and approximate text-vs-table consistency) were internally consistent within stated tolerances.

Checked items

1. ✔ **C1_L1_distance_from_sep** (Page 3, Sec. 2.2.2 (L1 point vicinity))
    - **Claim:** L1 point location approximated at $r_{\mathrm{L1}} = 0.536974 \times R_{\mathrm{sep}}$ with $R_{\mathrm{sep}} = 2000\ R_{\odot}$, placing L1 at approximately $1073.95\ R_{\odot}$.
    - **Checks:** multiplication_consistency
    - **Verdict:** PASS
    - **Notes:** Computed $0.536974 \times 2000 = 1073.948$ and compared to 1073.95.
2. ✔ **C2_stream_density_log_range_width** (Page 6, Sec. 3.1.1)
    - **Claim:** Mass Transfer Stream density spans $\log_{10}(\rho)$ from approximately -8.0 to -0.84 (code units).
    - **Checks:** range_width_recomputation
    - **Verdict:** PASS
    - **Notes:** Recomputed width $= 7.16$ and linear ratio$= 10^{7.16} \approx 1.445 \times 10^{7}$; no single reported width/ratio to compare against.
3. ✔ **C3_photosphere_vr_range_ordering** (Page 6, Sec. 3.1.2)
    - **Claim:** Photosphere radial velocity v_r ranges from approximately -0.99 to $+0.49$ (code units).
    - **Checks:** range_sanity_and_span
    - **Verdict:** PASS
    - **Notes:** Verified $v_{r,\mathrm{min}} < v_{r,\mathrm{max}}$; computed span $= 1.48$.
4. ✔ **C4_L1_vr_range_ordering** (Page 6, Sec. 3.1.2)
    - **Claim:** L1 Vicinity radial velocity v_r range is approximately -0.63 to -0.11 (code units).
    - **Checks:** range_sanity_and_span
    - **Verdict:** PASS
    - **Notes:** Verified $v_{r,\mathrm{min}} < v_{r,\mathrm{max}}$ and both endpoints are negative; computed span $= 0.52$.

5. ✔ **C5_feature_detection_top5pct_vs_95th** (Page 5 (Sec. 2.6) and Page 9 (Sec. 3.4))

   - **Claim:** Feature threshold described as 'top 5%' of $|\dot{m}|$ distribution (Methods) and also as 'exceeded the 95th percentile' (Results).
   - **Checks:** equivalence_of_percentile_statement
   - **Verdict:** PASS
   - **Notes:** Confirmed $(1 - 0.05) \times 100 = 95$.

6. ✔ **C6_structure_count_consistency** (Page 9, Sec. 3.4)

   - **Claim:** Identified 18 distinct structures of significant size (containing more than 10 grid cells).
   - **Checks:** inequality_interpretation_check
   - **Verdict:** PASS
   - **Notes:** Logical interpretation only: $' > 10$ cells' implies minimum included cells $= 11$; cannot verify count$= 18$ without component cell counts.

7. ✖ **C7_table1_volume_per_cell_consistency_across_structures** (Page 10, Table 1)

   - **Claim:** Table 1 reports num_cells and volume ($R_\odot^3$) for five structures; implied average volume per cell should be roughly constant if cell volumes are uniform/averaged.
   - **Checks:** derived_ratio_consistency
   - **Verdict:** FAIL
   - **Notes:** Computed volume/num_cells for the five structures and found max/min ≈ 42.20, exceeding the rel_tol-based threshold max/min $= 1.5$.

8. ✔ **C8_table1_mass_per_cell_outlier_check** (Page 10, Table 1)

   - **Claim:** Table 1 reports num_cells and total_mass for five structures; implied mass per cell can be compared to spot potential transcription errors (e.g., structure #10 very massive vs others).
   - **Checks:** derived_ratio_outlier_scan
   - **Verdict:** PASS
   - **Notes:** Computed mass/num_cells and compared to the median; no values exceeded the stated extreme outlier thresholds (ratio $> 10^3$ or $< 10^{-3}$).

9. ✔ **C9_structure10_mass_matches_text** (Page 9, Sec. 3.4 and Page 12, Conclusion point 4; also Table 1 (Page 10))

   - **Claim:** Structure #10 total mass is reported as 4.29×10^5 code units in Table 1 and reiterated as 4.29×10^5 code units in Conclusions.
   - **Checks:** repeated_value_match
   - **Verdict:** PASS

   - **Notes:** Exact match after parsing scientific notation.

10. ✔ **C10_structure10_rcom_matches_text** (Page 9, Sec. 3.4 and Table 1 (Page 10))
    - **Claim:** Structure #10 is said to be located at radial center of mass $r \approx 1725\ R_\odot$; Table 1 gives $r_\mathrm{com} = 1724.60\ R_\odot$.
    - **Checks:** approximate_value_consistency
    - **Verdict:** PASS
    - **Notes:** Difference consistent with rounding in the text (≈).

11. ✔ **C11_structure6_rcom_matches_text** (Page 10, immediately after Table 1)
    - **Claim:** Structure #6 propagated to $r \approx 4515\ R_\odot$; Table 1 gives $r_\mathrm{com} = 4515.17\ R_\odot$.
    - **Checks:** approximate_value_consistency
    - **Verdict:** PASS
    - **Notes:** Difference consistent with rounding in the text (≈).

12. ✔ **C12_anisotropy_ratio_bounds_claim** (Page 8, Sec. 3.3.1)
    - **Claim:** Anisotropy ratio $\lambda_\mathrm{max}/\mathbf{mean}(\lambda)$: 1 signifies isotropic; 3 represents a purely one-dimensional beam.
    - **Checks:** algebraic_identity_check
    - **Verdict:** PASS
    - **Notes:** Verified $\lambda_\mathrm{max}/\mathbf{mean}(\lambda) = 1$ for $[1, 1, 1]$ and $= 3$ for $[1, 0, 0]$.

13. ✔ **C13_alignment_angle_opposed_condition** (Page 8, Sec. 3.3.2 and Page 12, Key finding 3)
    - **Claim:** Angle greater than 90 degrees implies gas moving opposed to principal direction; peak alignment angle in stream ≈ 110.5 degrees (> 90).
    - **Checks:** inequality_check
    - **Verdict:** PASS
    - **Notes:** Checked $110.5 > 90$.

14. ✔ **C14_kmeans_elbow_k_matches_selected_k** (Page 10, Sec. 3.5)
    - **Claim:** Optimal number of clusters selected as $k = 5$; stated to be supported by pronounced elbow at $k = 5$.
    - **Checks:** repeated_constant_consistency
    - **Verdict:** PASS
    - **Notes:** Verified selected $k = 5$ lies within the stated tested range $k = 3$ to $k = 8$.

Limitations

- Only parsed text from the PDF pages was available; numerical arrays, histogram bins/counts, and correlation/eigenvalue outputs referenced as saved files are not included, preventing recomputation of most reported statistics.
- No plot digitization was performed; checks that would require reading values from figures were marked unverified.
- Several numeric claims are qualitative (e.g., 'broad', 'strong') or depend on dataset-derived quantities; only algebraic/logical consistency checks and table-derived ratio sanity checks were included as FAST-code candidates.